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Journal of Computational and Applied Mathematics 206 (2007) 358–365

JOURNAL OF  
COMPUTATIONAL AND  
APPLIED MATHEMATICS[www.elsevier.com/locate/cam](http://www.elsevier.com/locate/cam)

# Iterative algorithms for multi-valued inclusions with $Z$ mappings

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Received 13 March 2006; received in revised form 11 July 2006

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## Abstract

We consider a problem of solution of a multi-valued inclusion with  $Z$ -type mapping on a cone segment and propose extensions of Jacobi algorithms to find its solution. A modified double iteration process yields both upper and lower solutions. Some examples of applications are also described.

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MSC: 47J20; 47H07; 65K10

Keywords: Multi-valued inclusions;  $Z$ -mappings; Jacobi algorithms

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## 1. Introduction

The problem of solving a system of nonlinear equations is one of the basic and most investigated problems considered in Nonlinear Analysis, and many results have been derived in its theory and solution methods; see e.g., [12] and references therein. Most results are related to the single-valued and differentiable cases and are usually based upon the Newton-type methods. However, these conditions seem too restrictive for many applications arising in Mathematical Physics and Economics, which are formulated as multi-valued inclusions or variational inequalities; see e.g., [10,1]. One of the most efficient approaches to investigate such problems consists in exploiting order monotonicity properties of the underlying mapping, which enable one to obtain significant existence results even for multi-valued discontinuous mappings; see e.g., [1, Section 9.4], [11, Chapters 3–4]. But the development of solution methods is restricted by continuous single-valued mappings. This “gap” is caused by many difficulties. For instance, finding even suitable multi-valued order monotonicity concepts is not so easy.

Recently, in [5], a Jacobi-type algorithm for solving complementarity problems whose cost mappings are compositions of single-valued  $Z$ -mappings and multi-valued diagonal monotone mappings was proposed. In this paper, we intend to develop such an algorithm for multi-valued inclusions involving more general compositions of multi-valued mappings, thus extending the usual Jacobi algorithm from the single-valued case; see e.g., [12].

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doi:10.1016/j.cam.2006.07.025